GEOMETRY CONCEPTS IN ARCHITECTURAL DESIGN

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ABSTRACT: The relationship between geometry and architectural design are described and discussed along some examples. Geometry is the fundamental science of forms and their order. Geometric figures, forms and transformations build the material of architectural design. In the history of architecture geometric rules based on the ideas of proportions and symmetries formed fixed tools for architectural design. Proportions were analyzed in nature and found as general aesthetic categories across nature and art. Therefore proportions such as the golden section were seen as the power to create harmony in architecture as well as in art and music. According Pythagoras there were general principles for harmony. They were also applied in architecture and they found a further development especially in the renaissance. Leon Battista Alberti integrated such general harmonic proportion rules in his theory of architecture and realized them in his buildings. To find general principles of harmony in the world were the main research aims of Johannes Kepler in his "Harmonice mundi". These principles of harmony were based on geometry. Another important branch in the history of architectural design principles was the "golden section" or "divina proportione". "Modulor" of Le Corbusier is an example of an architectural design and formation concept based on the golden section. The concept of symmetry is combined with the idea of harmony and proportion. Symmetry operations are concerned with motions of figures and shapes. Geometry can be seen also as a structural science. The architectural design is based on geometric structures developed out of the idea of transformations. The symmetry transformations are visible as design concepts through history of architecture. In contemporary architecture there are no fixed rules about design concepts. But there are still relations to geometric space concepts. There is a need of new geometric background for architectural design. Examples of architecture and designing will be presented and discussed in their relationship to geometry. The role of geometry in architectural design processes will be analyzed exemplarily through history of architecture and new fruitful approaches show actual and future perspectives.

Keywords: Geometric structures, harmony, proportions, architectural design.

1. INTRODUCTION

As the fundamental science of forms and their order geometry contributes to the process of composition and designing in architecture. Composition in architecture starts with elements and their relations. Geometry is able to make a contribution to this process by dealing with geometric figures and forms as elements as well as proportions, angles and transformations as relations between them. Structures build the foundation of composing. Structures indicate general systems of order in various scientific disciplines, derived from the Latin notion "structura" which means join together in order. Mathematics can be seen as a general science of structures by considering systems of elements and their relations or operations. This concept is for example the background for the innovative approach to composition of Richard Buckminster Fuller. "Mathematics is the science of structure and pattern in general." [7]

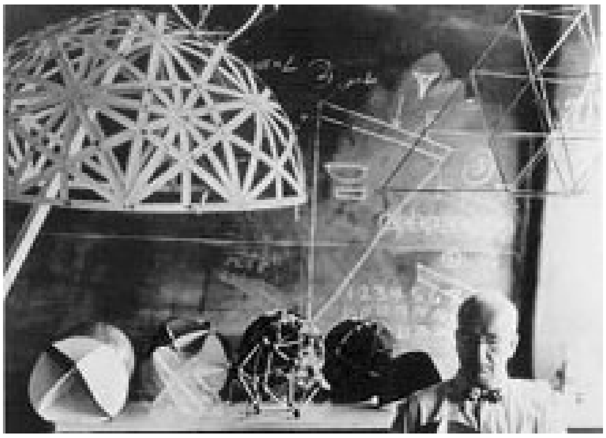

Figure 1: R. B. Fuller with models, 1949

In his research he developed for example a systematic way to subdivide the sphere. His structural thinking, starting with the Platonic Solids, led to the geodesic grids and finally his built geodesic domes. Geometry can be seen as the science to describe structures. Max Bill works in his art with geometric structures as processes, for example in his variations about a single theme, the process from triangle to octagon. With his variations he clarified his methods for generating artworks.

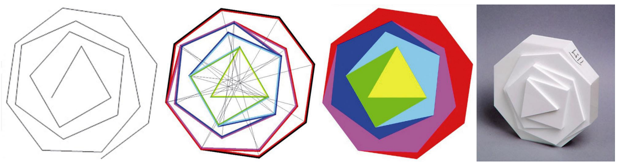

Figure 2: Max Bill, 1935-1938, Variations

Max Bill thought about the relationship between structures and art. In his opinion rhythmical order as the creative act of the artist produces an artwork starting with a general structure. Through history of geometry and architecture there were developed some rules based on geometry which formed the basis for architectural composition. In the following we will analyze the role of geometry in the architectural design processes through several examples along history of architecture.

2. HARMONY AS A PRINCIPLE OF COMPOSITION

The notion of harmony is seen as a fundamental principle of composition in history of architecture. Composition is based on harmony and order as aesthetic categories. The understanding of harmony is based on the mythological person "Harmonia", the goddess of harmony, who was seen as the daughter of Ares, the god of war, and Aphrodite, the goddess of love and beauty. Harmonia is the symbol of the union of antagonisms. Harmony means the connection of different or opposed things to an arranged whole. The antiquity science itself is conducted by principles of harmony and order.

2.1 Pythagoras

In the Pythagorean approach all occurrences are seen under a general principle. This principle wants to be a principle of composition by understanding all processes in mathematical orders. Arithmetic, geometry, astronomy and music, the sciences of Quadrivium are all based on this general principle. Pythagoras was convinced that harmony, all things and principles of being can be grasped by integers and mathematical regularities. He discovered that the music intervals form simple relations according the division of the string and the number of oscillations. The Tetraktys: numbers 1 to 4 (4 elements, 4 cardinal points) form the foundation according Pythagoras. The idea of harmonic proportions is a general principle for all sciences and applications.

2.2 Alberti

In reference to this antique understanding of harmony as the union of antagonisms Leon Battista Alberti (1404-1472) developed his principles of architecture. “De Re Aedificatoria” [1] is subdivided into ten books and describes how to achieve harmony in architecture. Beauty was for Alberti "the harmony of all parts in relation to one another," and subsequently based on the Pythagorean ideas "this concord is realized in a particular number, proportion, and arrangement demanded by harmony". Alberti’s ideas remained the classic treatise on architecture from the sixteenth until the eighteenth century and even longer.

2.3 Kepler

Harmony as a concept for all sciences and the whole world is also expressed in Johannes Kepler‘s “Harmonices mundi”. Johannes Kepler (1571-1630) well known as scientist, astronomer and mathematician based his harmony concept on geometry, especially the Platonic Solids. He was a Pythagorean mystic and considered mathematical relationships to be the fundament of all nature and creations. Geometrical concepts are in his theory the fundament of nature and science as well as art and music.

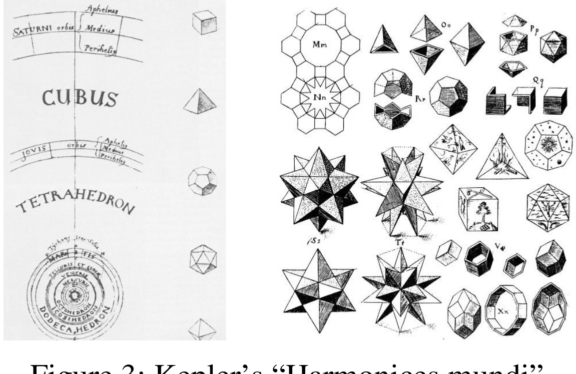

Figure 3: Kepler’s “Harmonices mundi”

Therefore also creation and design is based on the geometric world concept.

2.4 Golden Section

Such a fundamental principle of harmony derived from nature, applied in art, architecture and music can be seen in the golden section. The idea of the golden section shows the coherence of composition and geometry. This idea steps longtime through history of architecture. Hippasos of Metapont (450 B.C.) found it in his research about the pentagon and the relation of its edge length and the diagonal. Euclid (325-270 B.C.) was the first who described the golden section precisely also as a continuous division. In the following time golden section was seen as the ideal proportion and the epitome of esthetics and harmony. Especially in the renaissance, harmonic proportions were based on the geometric relations according the golden section in art, architecture as well as in music. Filippo Brunelleschi built Santa Maria del Fiore in Florence 1296 based on the golden section and the Fibonacci numbers.
The “Modulor” of Le Corbusier [5] is an example of an architectonic concept of designing and creating according geometric rules in modern architecture, but it remains bound to the classical conception of harmony.

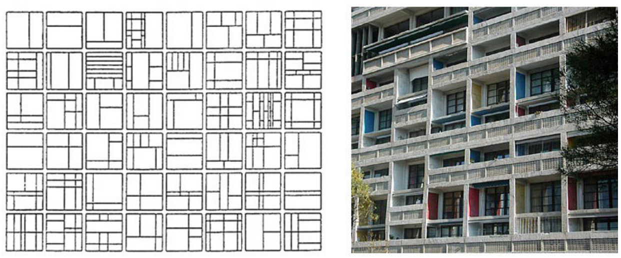

Figure 4: “Jeux de panneaux” and “Unité d’Habitation”, Le Corbusier

The structuring of the windows in Unité d’Habitation, Marseille, 1947 (Figure 4) shows various kinds of formations by maintaining the same structure principle subdividing according the golden section. A structural equivalence between music and architecture is obvious in

"Ondulatoires" in the music composition "Metastasis" by Xenakis and the façade of "La Tourette" by Le Corbusier/Xenakis 1952 [11] .
In his book "Architektur und Harmonie" Paul v. Naredi-Rainer [13] characterizes the relation between architecture and geometry: geometry has an important role for architecture in the process of form finding and form development without determining architecture exclusively.

3. SYMMETRY AND TRANSFORMATIONS

Another fundamental notion in the history of architecture is the concept of symmetry closely connected with the idea of harmony. "Symmetry", derived from the Greek "syn" which means together and "metron" which means measure, is understood as the harmony between the parts of an object and the way of the combination of several parts. Vitruv [15] described in this comprehension "Symmetria" as an important category of architecture. In this early understanding, symmetry describes the combination of the parts in a general way not in the mathematical meaning.
The mathematical symmetry concept of today was developed in the context of crystallography. Only in the middle of the 19^{th} century mathematicians became interested in the concept of symmetry. Later by introducing the concept of transformation in mathematics the concept of symmetry was changed and expanded. The "Erlangen Programme", 1872, by Felix Klein introduces geometry as the discipline of the invariants of transformation groups. Symmetry now is understood as the invariance under a certain kind of transformation [14].

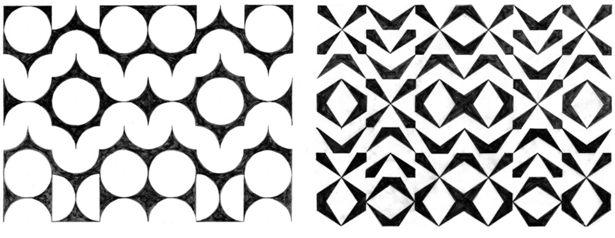

Figure 5: Two ornaments based on the same sequence of congruence transformations

Figure 5 shows a student work (Markus Weissenmayer) of creating patterns by congruence transformations. The students had to choose a start element and to develop a sequence of congruence transformations in a storyboard. Then the same storyboard is applied to another start element. This simple task tries to achieve an understanding of structural thinking.
The process of congruence transformations can be expressed in a clear way by the operations of folding and cutting. Folding gets more and more important for creating processes in architecture as well as in industry. A study about the geometric symmetry conditions of folding by a student of architecture (Eric Pigat) [12] and an example of industrial origami [17] are shown in Figure 6. The mathematical comprehension of symmetry as transformation processes only starts nowadays to influence the creating processes in architecture.

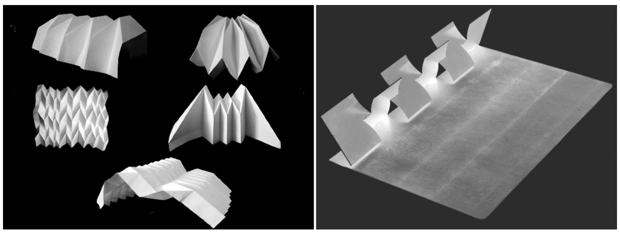

Figure 6: Folding study and industrial origami

4. GEOMETRIC AND ARCHITECTURAL SPACE CONCEPTS

The architectural space is based on a geometric space concept. Especially in the creation process architecture is thought in relation to a geometric space. Robin Evans [6] analyzes the relationship between geometry and architecture: "The first place anyone looks to find the geometry in architecture is in the shape of buildings, then perhaps the shape of the drawings of the buildings. These are the locations where geometry has been, on the whole, stolid and dormant. But geometry has been active in the space between and the space at either end." [6, p.xxxi].
According Evans, in history of architecture you find this misunderstanding of the role of geometry. In his historical study he refers to the

relations between Gaspard Monge's Descriptive Geometry and Jean-Nicolas Louis Durand's theory of architecture. Durand taught architecture at l'École Polytechnique in Paris at the same time as Monge around 1800. Durand developed an universal planning grid for architecture. Evans describes that Durand's grid architecture (Figure 7) is based on the misunderstanding of the spatial coordinate system. Instead of understanding the coordinate system in an abstract way, he transformed the coordinate planes directly in architecture as floor and walls.

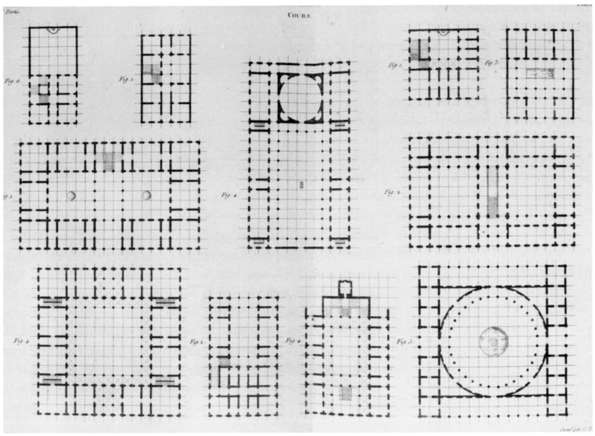

Figure 7: Durand's grid architecture, courtyards

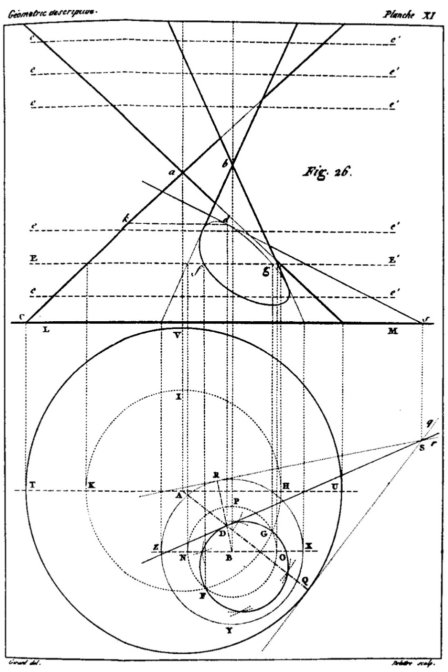

Figure 8: Intersection of two cones by Monge

At the same time the mathematicians were able to create curves and curved surfaces with the help of the coordinate system.

"… while descriptive geometry encouraged the free orientation of forms in relation to one another, Durand's orthographic projection was used to enforce the frontal and rectilinear. So whatever the defects in Durand's teaching, Monge's geometry can hardly be held to account for them." [6, p.327]. Also today you find this misunderstanding on the role of geometry in architecture if the strong forms are equated with geometry and the organic forms are seen in contrary to geometry. Evans states in his book appropriately: "When architects attempt to escape from the tyranny of geometry, meaning by the tyranny of the box, where can they escape to? Either they must give up geometry altogether (which would be exceptionally difficult), or they escape to another, always more complex and demanding geometry, or they do the last while giving the impression of having done the first, …" [6, p.331].

It is not the challenge of geometry to provide a catalogue of eidetic forms for architecture. Geometry rather provides the geometrical understanding of space as a background for architecture. Whereas the Euclidean geometry has its roots in measurements and therefore corresponds with the tactile space, the projective geometry corresponds with the visual space and refers to the perception. Evans pleads in involving projective geometry in architecture. He states that the indication of the two different kinds of geometry "enable us to see why architectural composition is such a peculiar enterprise: a metric organization judged optically, it mixes one kind of geometry with the other kind of assessment. Perhaps this is reason enough for the confusion surrounding it." [6, p.xxxiii]

The development of projective geometry got its impetus from architectural demands. The architect Brunellschi developed constructive principles for perspectives by using geometrical projection methods. Alberti summarized the results of the research in perspective to a teaching concept. Projective geometry originated from generalizing the use of vanishing points and constructing perspective drawings. Now according Evans there could be new impulses for architectural design by looking for the relationship between projection and architecture itself. The projections are operating between things and are seen as transitive relations. The diagram in Figure 9 shows four types of targets: designed object, orthographic projection, perspective and imagination combined with the perception of an observer. The diagram is thought by Evans as a tetrahedron, so that the center disappears and all relations are equitable. The routes between the targets can be traveled in either direction.

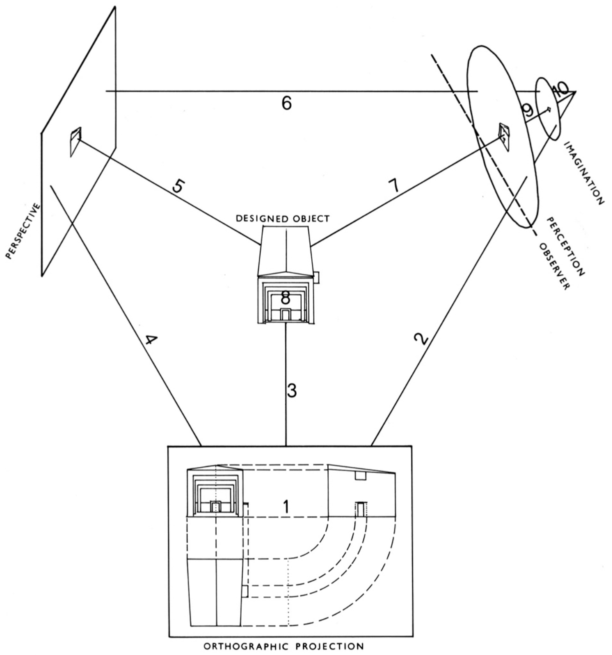

Figure 9: Diagram of projective transactions

The diagram of Evans can be interpreted as the sign model of the architectural design processes. Geometry is located in the mind, architecture in perceptible materialized reality. The diagram classifies the sign relations between the designed architectural object, the imagination and the drawings. “Between geometry and architecture we have somehow hopped from inside the mind to outside. So when dealing with architectural geometry, we seem to be dealing with this route or doorway between mental and real.” [6, p.354].

Such interactions between the described fields will be illustrated by some examples. Perspective for example affects back to the designed object. The classical example of an arcaded courtyard in “Palazzo Spada” in Rome, designed by Francesco Borromini 1635 (Figure 10), is an architectural trompe-l'œil in which diminishing rows of columns and a rising floor create the optical illusion of a long gallery.

Figure 10: Arcaded courtyard of “Palazzo Spada”, Rome, F. Borromini 1635 [18]

An actual example can be seen in the building “Phaeno Science Center” in Wolfsburg, Germany, built by Zaha Hadid in 2005 (Figure 11). An interacting between perspective and designed object can be often noticed in Zaha Hadid’s work.

Figure 11: Phaeno Science Center, Wolfsburg, Germany, built by Zaha Hadid, 2005 [19]

Preston Scott Cohen [4] works in his architectural design projects directly with the method of projection. He studied along historical examples that according principles of harmony symmetrical designed buildings get distorted by perspective projections in order to create a visual reality. Cohen develops a method using a "perspective apparatus" in the designing process. "The process of projection is reversible; perspectives may serve as objects and vice versa." [16]. He starts for example with a perspective drawing of an object. This perspective is then assumed to be an orthographic projection from which other views are later derived to produce a third dimension and finally to create the object.

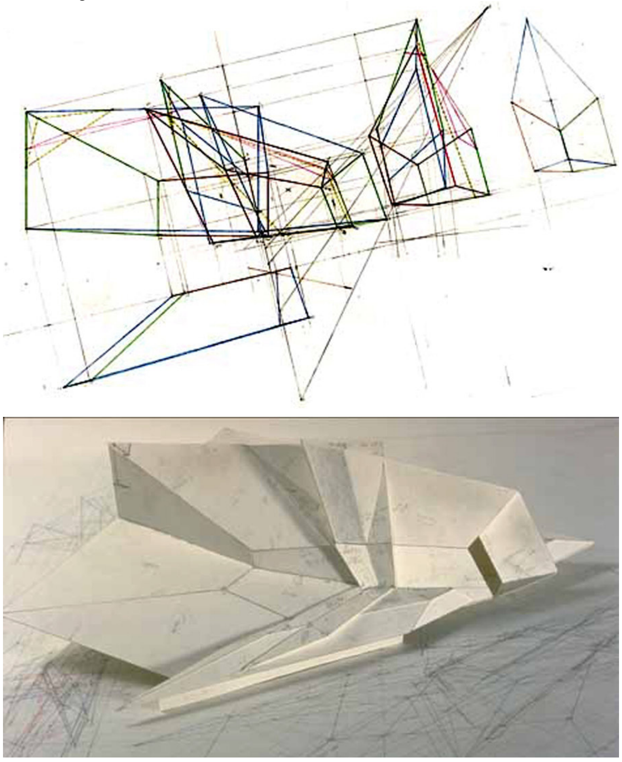

Figure 12: Patterns for Head Start Facilities, drawing and model, Cohen, 1994 [16]

Cohen uses the method of perspective projection to create architecture. His ideas can be located in Evans diagram (Figure 9) between the fields 3, 4 and 5. It is an interesting new example of architectural design where geometry leads the design process according the idea of projection.

Another example for such a doorway between mental and real may be seen in some works of Ben van Berkel and Caroline Bos when they apply topological thinking to architecture. In "Move 1" they write: "Blob or box - it doesn't matter anymore. (...) Do you like boxes? No problem, nowadays your shoes can be packed in a box or a bag; you can put the box in the bag too - it's up to you." [2, p.221]. The transfer from box to blob may be realized by topological symmetry where the connectedness of an object remains. In the Moebius house for example the idea of the Moebius band leads the design process, not in a figurative but in a conceptualized sense, as they write in Move 2: "The mathematical model of the Moebius band is not literally transferred to the building, but is conceptualised or thematised and can be found in architectural ingredients, such as the light, the staircase and the way in which people move through the house." [2, p.43].

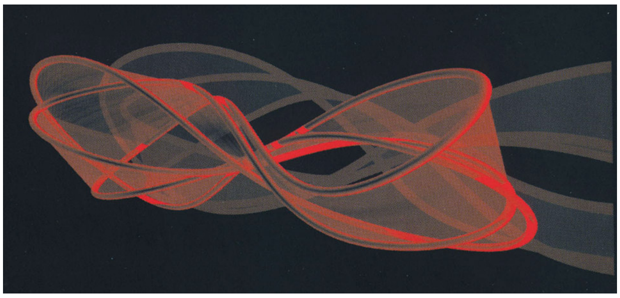

Figure 13: Moebius band concept

Figure 14: Moebius house, Berkel, 1993-98

The examples show various ways to refer to geometry in the architectural processes also in present time. It seems that there exists a fruitful relationship between geometry and architecture in the past as well as today although the role of geometry is sometimes only seen as a past historical one. It takes some time until geometric new developments are picked up by architecture even though architecture leads the way for mathematics in the renaissance time. It starts now that the mathematical ideas of transformation geometry, projective geometry, non-Euclidean geometry or topology of the 19^{th} century find their correspondent in architecture. Perhaps a new powerful relationship of geometry and architectural design begins.

5. CONCLUSIONS

The relationship between architectural design and geometry starts with the notion of harmony as the principle for all sciences and creations. The analysis of the antique comprehension of harmony shows the geometrical root and the superior idea of this concept for all sciences and designing disciplines. Today the various sciences and arts are in most cases strongly separated. Therefore there is the risk that the powerful relationship between geometry and architecture gets lost. Steven Holl, who refers in his architectural work to geometry and other sciences, noticed: "For example Johannes Kepler's Mysterium Cosmographicum united art, science, and cosmology. Today, specialization segregates the fields; yawning gaps prohibit potential cross-fertilization." [8].

By remembering the historical relations between geometry and architectural design we help to keep the background of our culture but also to understand the fruitful combination between geometrical thinking and architectural designing. By integrating experiments on using geometric structures for designing in the architecture curriculum we should reflect this relationship and try to develop new impulses for geometrical based designing in architecture. Only few examples were shown here in an overview. There are more efforts necessary in the future to work out this relationship in detail, historical and theoretical, from an architectural and a geometrical point of view as well as to experience and apply it in the practice of architectural design.

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